Covariant Level-Classification Scheme and Chiral Symmetry

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Starting from the bi-local Klein-Gordon Equation with spin-independent squared-mass operator, we give a covariant quark representation of general composite meson systems with definite Lorentz transformation properties. For benefit of this representation we are able to deduce automatically the transformation rules of composite mesons for general symmetry operations from those of constituent (exciton) quarks. Applying this we investigate especially physical implication of chiral symmetry for the meson systems, and present a covariant level-classification scheme, leading to a possible existence of new meson multiplets of "chiralons."

§1. Introduction

There are the two contrasting view points of composite quark-antiquark mesons: The one is non-relativistic, based on the approximate symmetry of LS-coupling in the non-relativistic quark model (NRQM); while the other is relativistic, based on the dynamically broken chiral symmetry typically displayed in the Nambu Jona-Lasinio (NJL) model. The π -meson (or π -nonet) is now widely believed to have a dual nature of non-relativistic system and also of relativistic system as a Nambu-Goldston (NG) boson in the case of spontaneous breaking of chiral symmetry. However, no successful attempts to unify the above two view points have been yet proposed.

On the other hand we have developed the covariant oscillator quark model $(COQM)^{1), 2), 3)$ for many years as a covariant extension of NRQM, which is based on the boosted LS-coupling scheme. The meson wave functions (WF) in COQM are the restricted tensors in the $\tilde{U}(4) \otimes O(3,1)$ space which reduce at the rest frame to those in the $SU(2)_{\rm spin} \otimes O(3)_{\rm orbit}$ space in NRQM. Although the COQM had been successfully applied to various non-static as well as static problems of hadrons with general quark configuration, no consideration on chiral symmetry was given there.

The purpose of this paper is to get rid of this defect in COQM and is to give a unified view point of the two contrasting ones of the composite meson systems, extending the WF to the general tensors in the $\tilde{U}(4) \otimes O(3,1)$ space, which are required for taking into account chiral symmetry.

§2. Covariant Framework for Describing Composite Mesons

For meson WF described by $\Phi_A{}^B(x_1, x_2)$ (x_1, x_2) denoting the space-time coordinate and $A = (\alpha, a)(B = (\beta, b))$ denoting the Dirac spinor and flavor indices of

constituent quark (anti-quark)) we set up the bilocal Yukawa equation ³⁾

$$\left[\frac{\partial^2}{\partial X_{\mu}^2} - \mathcal{M}^2(x_{\mu}, \frac{\partial}{\partial x_{\mu}})\right] \Phi_A{}^B(X, x) = 0 \tag{2.1}$$

(X(x)) denoting the center of mass (CM) (relative) coordinate of meson), where the \mathcal{M}^2 is squared mass operator including only a central, Dirac-spinor-independent confining potential. The WF is separated into the plane wave describing CM motion and the (Fierz-component) internal WF as

$$\Phi_A{}^B(x_1, x_2) = \sum_{\mathbf{P}_n, n} (e^{iP_n X} \Psi_{n, A}{}^{(+)B}(x, P_n) + e^{-iP_n X} \Psi_{n, A}{}^{(-)B}(x, P_n)), (2.2)$$

where the Fierz components $\Psi_n^{(\pm)}$ are eigenfunctions of \mathcal{M}^2 ; $P_{n,\mu}^2 = -M_n^2$, $P_{n,0} = \sqrt{M_n^2 + \mathbf{P}_n^2}$; and the label (\pm) represents the positive (negative) frequency part; and n does a freedom of excitation. We have the following field theoretical expression for the WF in mind as a guide for the present semi-phenomenological approach:

$$\Phi_A{}^B(x_1, x_2) = \sum_n [\langle 0 | \psi_A(x_1) \bar{\psi}^B(x_2) | M_n \rangle + \langle M_n^c | \psi_A(x_1) \bar{\psi}^B(x_2) | 0 \rangle], \quad (2.3)$$

where $\psi_A(\bar{\psi}^B)$ denotes the quark field (its Pauli-conjugate) and $|M_n\rangle$ ($\langle M_n^c|$) does the composite meson (its charge conjugate) state. The internal WF is expanded in terms of a complete set $\{W_i\}$ of free bi-Dirac spinors of quarks and anti-quarks as

$$\Psi_A^{(\pm)B}(x, P_n) = \sum_i W_{i\alpha}^{(\pm)\beta}(P_n) \phi_a^{(\pm)b}(x, P_n). \tag{2.4}$$

§3. Complete Set of Spin Wave Function and Irreducible Meson Components

We set up the conventional "free" Dirac quark-spinors with four-momentum of composite meson itself $P=P_M,\ u_q(P_\mu,s_q),$ which satisfies the Dirac equation, $(iP\cdot\gamma+M)u_q(P_\mu,s_q)=0$, where s_q represents the spin up-down of the quark and $P_0>0$ is the energy of the meson. In §4 we consider the chiral transformation property of composite meson systems. Through chiral transformation, $u_q(P_\mu,s_q)$ is related with $\gamma_5\ u_q(P_\mu,s_q),$ which has the energy-momentum $-P_\mu$, since $\gamma_5\ u_q(P_\mu,s_q)$ satisfies the Dirac equation $(-iP\cdot\gamma+M)\ \gamma_5\ u_q(P_\mu,s_q)=0$. Thus, $\gamma_5\ u_q(P_\mu,s_q)\sim u_q(-P_\mu,s_q)$. We define $(u_q(P_\mu,s_q),u_q(-P_\mu,s_q)),$ both of which are the solutions of the above Dirac equation, as $(u_q(P_\mu,s_q),u_q(-P_\mu,s_q))=(u(P,s_q),-s_qv(P,-s_q)).$

Conventional "free" Dirac antiquark-spinors with four-momentum P_{μ} , $\bar{v}^{\bar{q}}(P_{\mu}, s_{\bar{q}})$, satisfies the Dirac equation, $\bar{v}^{\bar{q}}(P_{\mu}, s_{\bar{q}})(-iP \cdot \gamma + M) = 0$, where $s_{\bar{q}}$ represents the spin up-down of the antiquark. We define $(\bar{v}^{\bar{q}}(P_{\mu}, s_{\bar{q}}), \bar{v}^{\bar{q}}(-P_{\mu}, s_{\bar{q}}))$, both of which are the solutions of the above Dirac equation, as $(\bar{v}^{\bar{q}}(P_{\mu}, s_{\bar{q}}), \bar{v}^{\bar{q}}(-P_{\mu}, s_{\bar{q}})) = (\bar{v}(P, s_{\bar{q}}), s_{\bar{q}}\bar{u}(P, -s_{\bar{q}}))$.

It is to be noted that all four spinors for both "quarks and anti-quarks" are necessary to describe the spin WF of mesons. Then the complete set of bi-Dirac spinors $W^{(+)}(P)$ is given by

$$U(P) = u_{q}(p_{1}, s_{q}) \bar{v}^{\bar{q}}(p_{2}, s_{\bar{q}})|_{p_{i,\mu} = \kappa_{i} P_{\mu}} = u(\mathbf{P}, s_{q}) \bar{v}(\mathbf{P}, s_{\bar{q}}),$$

$$C(P) = u_{q}(p_{1}, s_{q}) \bar{v}^{\bar{q}}(-p_{2}, s_{\bar{q}})|_{p_{i,\mu} = \kappa_{i} P_{\mu}} = u(\mathbf{P}, s_{q}) \bar{u}(\mathbf{P}, -s_{\bar{q}}) s_{\bar{q}},$$

$$D(P) = u_{q}(-p_{1}, s_{q}) \bar{v}^{\bar{q}}(p_{2}, s_{\bar{q}})|_{p_{i,\mu} = \kappa_{i} P_{\mu}} = -s_{q} v(\mathbf{P}, -s_{q}) \bar{v}(\mathbf{P}, s_{\bar{q}}),$$

$$V(P) = u_{q}(-p_{1}, s_{q}) \bar{v}^{\bar{q}}(-p_{2}, s_{\bar{q}})|_{p_{i,\mu} = \kappa_{i} P_{\mu}} = -s_{q} v(\mathbf{P}, -s_{q}) \bar{u}(\mathbf{P}, -s_{\bar{q}}) s_{\bar{q}},$$

$$(3.1)$$

where in Eq.(3·1) we have defined technically the momenta of "constituent quarks" * as

$$p_{i,\mu} \equiv \kappa_i P_{\mu}, \ p_{i,\mu}^2 = -m_i^2; \ P_{\mu}^2 = -M^2, M = m_1 + m_2$$

 $(\kappa_{1,2} \equiv m_{1,2}/(m_1 + m_2); \ \kappa_1 + \kappa_2 = 1).$ (3.2)

The respective members in Eq.(3·1), Non-Relativistic, \bar{q} -type and q-type Semi-Relativistic, and Extremely Relativistic ones, are expressed in terms of their irreducible composite meson WF as follows:

$$U_{A}{}^{B}(P) = \frac{1}{2\sqrt{2}} [(i\gamma_{5}P_{s,a}^{(NR)b}(P) + i\gamma_{\mu}V_{\mu,a}^{(NR)b}(P))(1 + \frac{iP \cdot \gamma}{M})]_{\alpha}{}^{\beta},$$

$$C_{A}{}^{B}(P) = \frac{1}{2\sqrt{2}} [(S_{a}^{(\bar{q})b}(P) + i\gamma_{5}\gamma_{\mu}A_{\mu,a}^{(\bar{q})b}(P))(1 - \frac{iP \cdot \gamma}{M})]_{\alpha}{}^{\beta},$$

$$D_{A}{}^{B}(P) = \frac{1}{2\sqrt{2}} [(S_{a}^{(q)b}(P) + i\gamma_{5}\gamma_{\mu}A_{\mu,a}^{(q)b}(P))(1 + \frac{iP \cdot \gamma}{M})]_{\alpha}{}^{\beta},$$

$$V_{A}{}^{B}(P) = \frac{1}{2\sqrt{2}} [(i\gamma_{5}P_{s,a}^{(ER)b}(P) + i\gamma_{\mu}V_{\mu,a}^{(ER)b}(P))(1 - \frac{iP\gamma}{M})]_{\alpha}{}^{\beta},$$

$$(3.3)$$

where all vector and axial-vector mesons satisfy the Lorentz conditions, $P_{\mu}V_{\mu}(P) = P_{\mu}A_{\mu}(P) = 0$. Here it is to be noted that, in each type of the above members, the number of freedom counted both in the quark representation and in the meson representation is equal, as it should be $(2 \times 2 = 4 \text{ and } 1 + 3 = 4, \text{ respectively})$.

§4. Level Classification and Chiral Symmetry

4.1. Level structure of ground state mesons

Thus far we have presented a general covariant kinematical framework for describing the (ground states of) composite meson systems. However, what kinds of mesons do really exist or not, that is, the meson spectroscopy, is a dynamical (still unsolved) problem of QCD.

For this problem, we follow a physical idea of dynamically broken chiral symmetry of QCD, typically displayed in the NJL model: In the heavy quarkonium $(Q\bar{Q})$ system both quarks(Q) and antiquarks (\bar{Q}) are possible to do, since $m_Q > \Lambda_{\text{conf}}$, only non-relativistic motions with positive energy, and the LS-symmetry is good.

^{*} In so far as concerned with Eqs. (3·1) and (3·2) the quantities κ_i and accordingly m_i are arbitrary and have no physical meaning. However, m_i have proved to be the effective masses of constituent quarks through the phenomenological applications of COQM so far made.

Table I. Level structure of the ground states of general quark meson systems: In $Q\bar{Q}$ -meson system the approximate LS-symmetry is expected to be valid, since m_Q and $m_{\bar{Q}} > \Lambda_{\rm conf}$: In $Q\bar{q}(q\bar{Q})$ -meson system the approximate chiral symmetry for light anti-quark (quark) $\bar{q}(q)$ is to be valid, since $m_{\bar{q}}(m_q) \ll \Lambda_{\rm conf}$; while the conventional heavy quark symmetry (HQS) is to be valid; In $q\bar{q}$ system the approximate chiral symmetry is to be valid.

Config.	Mass	Approx.Sym.	Spin WF	Meson Type
$Qar{Q}$	$m_Q + m_{\bar{Q}}$	LS sym.	$u_Q(p_\mu)\bar{v}^Q(p_\mu) = u(p)\bar{v}(p)$	P_s, V_μ
$qar{Q}$	$m_q + m_{\bar{Q}}$	q:chiral sym.	$u_q(p_\mu)\bar{v}^Q(p_\mu) = u(p)\bar{v}(p)$	P_s, V_μ
		\bar{Q} :HQ Sym.	$u_q(-p_\mu)\bar{v}^{\bar{Q}}(p_\mu) = -v(p)\bar{v}(p)$	S,A_{μ}
$Q\bar{q}$	$m_Q + m_{\bar{q}}$	Q:HQ Sym.	$u_Q(p_\mu) ar{v}_{ar{q}}^{ar{q}}(p_\mu) = u(p) ar{v}^{ar{q}}(p)$	P_s, V_μ
		\bar{q} :chiral sym.	$u_Q(p_\mu)\bar{v}^{\bar{q}}(-p_\mu) = u(p)\bar{u}(p)$	S, A_{μ}
$qar{q}$	$m_q + m_{\bar{q}}$	chiral sym.	$\frac{1}{\sqrt{2}}(u_q(p_\mu)\bar{v}^{\bar{q}}(p_\mu) \pm u_q(-p_\mu)\bar{v}^{\bar{q}}(-p_\mu))$	$P_s^{(N)}, V_{\mu}^{(N)};$
			$\sim \frac{1}{\sqrt{2}}(u(p)\bar{v}(p) \pm v(p)\bar{u}(p))$	$P_s^{(E)}, V_\mu^{(E)}$
			$\frac{(1,-i)}{\sqrt{2}}(u_q(p_\mu)\bar{v}^{\bar{q}}(-p_\mu)\pm u_q(-p_\mu)\bar{v}^{\bar{q}}(p_\mu))$	$S^{(N)}, A^{(N)}_{\mu};$
			$\sim \frac{(1,-i)}{\sqrt{2}}(u(p)\bar{u}(p)\pm v(p)\bar{v}(p))$	$S^{(E)}, A_{\mu}^{(E)}$

Accordingly the bi-spinor U is considered to be applied to $Q\bar{Q}$ system as a covariant spin WF. In the heavy-light quark meson $Q\bar{q}(q\bar{Q})$ system the $\bar{q}(q)$ make, since $m_q \ll \Lambda_{\rm conf}$, relativistic motions both with positive and negative energies, and the chiral symmetry concerning $\bar{q}(q)$ is good. Accordingly both the U and C (U and D) are to be applied to the $Q\bar{q}(q\bar{Q})$ system, leading to possible existence of new composite scalar and axial-vector mesons(see Eq.(3·3)). In this meson system it is to be noted that the conventional heavy quark symmetry (HQS) is also valid as an approximate symmetry. In the light quark $q\bar{q}$ -meson system both quarks q and anti-quarks q make, since $m_q \ll \Lambda_{\rm conf}$, relativistic motion with both positive and negative energies, and chiral symmetry is good. Accordingly the linear combinations of U and V are applied to the $q\bar{q}$ -system, and in this system there is a possiblity of existence of an extra(, in addition, to a normal) set of pseudo-scalar and vector mesons. Furthermore, the linear combinations of C and D are also applied, and normal and extra sets of composite scalar and axial-vector mesons possibly exist as relativistic S-wave bound states.

In the above discussion we assume that $\Lambda_{\rm conf} \sim 1 {\rm GeV}$ regardless of quark-flavor. The above expected level structure of ground states is summarized in Table I.

4.2. Level structure of excited state mesons

In classifying the excited-state mesons we can proceed essentially similarly as the case of ground state mesons. In the present approximation the masses of N-th excited states are given by $M_N^2 = M_G^2 + N\Omega$ ($M_G \equiv M_0$, Ω being the inverse Regge slope), and their spin wave functions are defined by the same formulas as given in §3 with substitution of constituent exciton-quark mass m_i by

$$m_i^* = \gamma_N \ m_i \ (\gamma_N \equiv M_N/M_G).$$
 (4.1)

The value of m_i^* for the $(q\bar{q})$ systems obtained by the formula (4·1) are seen $m^* \ll \Lambda_{\rm conf}$ for the lower levels, especially for the ground and first-excited states, and

accordingly we can expect that chiral symmetry for these states may be still good. Similarly, in the $(q\bar{Q})/(Q\bar{q})$ systems, the chiral symmetry concerning the light quark is also good for the several lower levels.

4.3. Expected spectroscopy of mesons

In our fundamental equation Eq.(2·1) the squared-mass spectra \mathcal{M}^2 is, as a first step in the pure-confining force limit, assumed to be Dirac-spinor independent. Actually we must take into account the various effects due to one-gluon-exchange potential and so on.

light – quark $(q\bar{q})$ meson system: In the present approximation all the ground state mesons expected in §3, $P_s^{(N,E)}$, $V_\mu^{(N,E)}$, $S^{(N,E)}$ and $A_\mu^{(N,E)}$ have the mass, $M_0 = m_1 + m_2$. Actually the mass of $P_s^{(N)}$, assigned to π -nonet, should be exceptionally low because of its nature* as a Nambu-Goldstone boson. The masses of the $V_\mu^{(N)}$ -nonet, assigned to ρ -meson nonet, are known to be almost equal to $M_0 = m_1 + m_2$. The masses of all the other ground-state mesons are expected to be almost equal to those of the $V_\mu^{(N)}$ -nonet, and to be lower than those of the first-excited states.

For the first-excited states, the chiral symmetry is expected to be still effective, so we expect the existence of a series of the first excited P-wave states of the ground state multiplets. They have the masses, which are almost equal to the first excited states of $V_{\mu}^{(N)}$ -mesons and lower than the second excited states of them. Among the multiplets newly predicted in the present scheme, to be called "chiralons," the especially interesting mesons are the ones with the exotic quantum numbers; $J^{PC} = 0^{+-}(S^{(E)}(S-wave)), \ 0^{--}(A_{\mu}^{(N)}(P-wave)), \ 1^{-+}(S^{(E)}(P-wave))$ and $1^{-+}(A_{\mu}^{(E)}(P-wave))$. Their masses, by the above mentioned estimate, is expected to be, respectively, $m(0^{+-}) \lesssim 1.3$ GeV and 1.3 GeV $\simeq m(0^{--}) \simeq m(1^{-+}) \simeq m(1^{-+}) \lesssim 1.7$ GeV.

<u>heavy - light quark ($Q\bar{q}$ and $q\bar{Q}$) meson system:</u> As is seen from Table I we expect the existence of new S and A_{μ} multiplets (at least the ground states). <u>heavy quark ($Q\bar{Q}$) meson system:</u> No new multiplets are expected to exist.

§5. Experimental Evidences and Concluding Remarks

In this paper we have presented a kinematical framework for describing covariantly the ground states as well as excited states of light-through-heavy quark mesons.

For light-quark mesons our scheme gives a theoretical basis to classify the composite meson systems unifying the two contrasting viewpoints based on NRQM with LS symmetry and on NJL model with chiral symmetry. The essential physical assumption is to set up the bilocal Klein-Gordon equation (Yukawa equation) with the squared-mass operator, which is, in the pure-confining force limit, independent of Dirac-spinor suffix, and accordingly is chiral symmetric. As a result is pointed out a possibility of existence of rather an abundant new nonets, chiralons, with masses

^{*} The $q\bar{q}$ -condensation are possible for the $S^{(N)}$, and $P_s^{(N)}$ other than $P_s^{(E)}$ is a NG boson.

lower than about 2 GeV; several ground and excited-state meson nonets.

For heavy/light quark meson systems we have similarly pointed out a possibility of existence of new multiplets(triplets), chiralons.

Presently we can give a few experimental candidates for the predicted new multiplets: One of the most important and interesting ones is the scalar σ nonet (the members are $\sigma(600)$, $^{4)}$ $\kappa(900)$, $^{4)}$ $a_0(980)$ and $f_0(980)$) which constitutes, $^{5)}$ with π -nonet, a linear representation of the chiral SU(3) symmetry. It is notable that the σ nonet is the relativistic S-wave state, which should be discriminated from the 3P_0 state.

Now the existence of three pseudoscalars ⁶⁾ with mass between 1 GeV~1.5GeV ($\eta(1295)$, $\eta(1420)$ and $\eta(1460)$) seems to be an established experimental fact. The two out of them may belong to the radially excited π -nonet, while the one extra to the ground states of the $P_s^{(E)}$ -nonet newly predicted.

Also we have the other candidates for chiralons: Recently it seems that the existence of two exotic particles $^{7)}$ in the $\eta\pi$ system with $J^{PC}=1^{-+}$ and with a mass around 1.5 GeV $\pi_1(1400)$ and $\pi_1(1600)$ has been accepted widely. These two particles have the mass in the region estimated in §4.3 to be assigned as the respective excited P-wave states of $S^{(E)}$ and $A^{(E)}_{\mu}$.

We have the longstanding problem in hadron spectroscopy: the mass and width of $a_1(1260)$ seem to be variant ⁸⁾ depending on the production process and/or decay channel. We have made ⁹⁾ recently a preliminary analysis of the data obtained by GAMS group WA102 experiment on process $\pi^-p \to 3\pi^0n$, leading to an evidence of existence of two $a_1(J^{PC}=1^{++})$ particles: $a_1^c(m=1.0\text{GeV})$, and $a_1^N(m=1.3\text{GeV})$ (to be assigned, respectively, to $A_{\mu}^{(N)}(^3S_1)$, and to the conventional a_1 particle).

Finally we should like to refer to a preliminary result of analysis on the heavy/light quark meson systems that a scalar chiralon B_0^{χ} with $M \simeq 5.52 {\rm GeV}$ may be observed ¹⁰⁾ in the $B\pi$ channel.

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